

Functional Programming

Specifics

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Operator

- › One or more symbols

```
! # $ % & * + . / < > ? @ ^ | - ~ = \ :
```

- › all are binary and infix
except unary minus: $(- x) \equiv \text{negate } x$.

Operators

```
a *** b = a ^ 2 + b ^ 2  
(***) a b = a ^ 2 + b ^ 2
```

Fucntions

```
f a b = a ^ 2 + b ^ 2  
a `f` b = a ^ 2 + b ^ 2
```

Associativity and Priority

```

infixl 9 !!
infixr 9 .
infixr 8 ^, ^^, **
infixl 7 *, /, `quot`, `rem`,
        `div`, `mod`

infixl 6 +, -
infixr 5 ++, :
infix 4 ==, /=, <, <=, >=, >,
        `elem`, `notElem`

infixr 3 &&
infixr 2 ||
infixl 1 >>, >>=
infixr 1 ==<<
infixr 0 $, $!, `seq`
    
```

Left section

```
(1 ***) ≡ (***) 1 ≡ \x -> 1***x
```

Function application

- > Function application
highest priority, a-la 10
- > \$ application

```

infixr 0 $
f $ x = f x
f (g x (h y)) ≡ f $ g x $ h y
    
```

- > Function composition

```

infixr 9 .
f . g = \x -> f (g x)

f(g(h(e(x)))) ≡
f . g . h . e $ x
    
```

Right section

```
(*** 1) ≡ \x -> x *** 1
```

Lazy evaluation (call-by-need)

- Delay the evaluation of an expression *until its value is needed*
- Also, try to avoid repeated evaluations

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Lazy lists

```
ghci> let a = [1..]
ghci> let ones = 1 : ones
ghci> ones
[1,1,1,1,1,1,,1,1,1,Interrupted.
ghci> take 5 ones
[1,1,1,1,1]
```

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```

Strict lists

```
data List a = Nil | !a :!(List a)
ghci> let onesS = 1 :! onesS
ghci> ones
^CInterrupted.
ghci> take 5 ones
^CInterrupted.
```

Lazy evaluation (call-by-need)

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Lazy lists

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```

Strict lists

```
data List a = Nil | !a :! !(List a)
ghci> let onesS = 1 :! onesS
ghci> ones
^CInterrupted.
ghci> take 5 ones
^CInterrupted.
```

A step back

How many values of type Bool?

```
data Bool = True | False
```

Lazy evaluation (call-by-need)

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Lazy lists

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A step back

How many values of type Bool?

```
data Bool = True | False
```

Hm ... **??? 3 ???** (in some sense)

```
bot :: Bool
bot = not bot
```


Lazy evaluation (call-by-need)

- › Delay the evaluation of an expression *until its value is needed*
- › Also, try to avoid repeated evaluations

Lazy lists

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ghci> ones
^CInterrupted.
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```

A step back

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```

Hm ... **??? 3 ???** (in some sense)

```
bot :: Bool
bot = not bot
```

In Haskell's *static* semantics its value is \perp

```
 $\perp$  :: forall {a} . a
```

undefined

```
ghci> undefined
*** Exception: Prelude.undefined
ghci> :t undefined
undefined :: forall {a} . a

f x y = x
ghci> f 42 undefined
42
```

Strict?

undefined

```
ghci> undefined
*** Exception: Prelude.undefined
ghci> :t undefined
undefined :: forall {a} . a

f x y = x
ghci> f 42 undefined
42
```

We could define seq as follows:

```
seq ⊥ b = ⊥
seq a b = b, if a ≠ ⊥

ghci> seq undefined 42
*** Exception: Prelude.undefined
ghci> seq (id undefined) 42
*** Exception: Prelude.undefined
```

Force!: seq

```
:info seq
seq :: a -> b -> b
  -- Defined in 'GHC.Prim'
infixr 0 `seq`
```

Strict?

undefined

```
ghci> undefined
*** Exception: Prelude.undefined
ghci> :t undefined
undefined :: forall {a} . a

f x y = x
ghci> f 42 undefined
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*** Exception: Prelude.undefined
```

up to weak WHNF!

```
ghci> seq (undefined,undefined) 42
42
ghci> seq (\x -> undefined) 42
42
ghci> seq ((+) undefined) 42
42
```

Why to force?

call-by-value

```
infixr 0 $!  
($!) :: (a -> b) -> a -> b  
f $! x = x `seq` f x
```

```
ghci> const 42 $! undefined  
*** Exception: Prelude.undefined
```

> Note: \$ and \$! has the same type

```
ghci> :t ($!)  
($!) :: (a -> b) -> a -> b  
ghci> :t ($)  
($) :: (a -> b) -> a -> b
```

Why to force?

call-by-value

```
infixr 0 $!  
($!) :: (a -> b) -> a -> b  
f $! x = x `seq` f x
```

```
ghci> const 42 $! undefined  
*** Exception: Prelude.undefined
```

> Note: \$ and \$! has the same type

```
ghci> :t ($!)  
($!) :: (a -> b) -> a -> b  
ghci> :t ($)  
($) :: (a -> b) -> a -> b
```

Do we need to force computations?

```
factorial = helper 1 where  
  helper acc k | k > 1 = helper (acc * k) (k - 1)  
               | otherwise = acc
```

Why to force?

call-by-value

```
infixr 0 $!  
($!) :: (a -> b) -> a -> b  
f $! x = x `seq` f x
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ghci> const 42 $! undefined  
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```
ghci> :t ($!)  
($!) :: (a -> b) -> a -> b  
ghci> :t ($)  
($) :: (a -> b) -> a -> b
```

Do we need to force computations?

```
factorial = helper 1 where  
  helper acc k | k > 1 = helper (acc * k) (k - 1)  
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```

What acc really stores?

Why to force?

call-by-value

```
infixr 0 $!  
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f $! x = x `seq` f x
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ghci> const 42 $! undefined  
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> Note: \$ and \$! has the same type

```
ghci> :t ($!)  
($!) :: (a -> b) -> a -> b  
ghci> :t ($)  
($) :: (a -> b) -> a -> b
```

Do we need to force computations?

```
factorial = helper 1 where  
  helper acc k | k > 1 = helper (acc * k) (k - 1)  
               | otherwise = acc
```

What acc really stores? (...((1 * n) * (n - 1)) * (n - 2) * ... * 2)

```
factorial = helper 1 where  
  helper acc k | k > 1 = (helper $! acc * k) (k - 1)  
               | otherwise = acc
```


A little more on forcing

Example

```
-- module Data.Complex
infix 6 :+:
data Complex a = !a :+: !a
```

```
ghci> case (1,undefined) of (_,_) -> 42
42
```

```
ghci> case 1 :+: undefined of _ :+: _ -> 42
*** Exception: Prelude.undefined
```

```
ghci> case 1 :+: undefined of _ -> 42
42
```

A little more on forcing

Example

```
-- module Data.Complex
infix 6 :+:
data Complex a = !a :+: !a
```

```
ghci> case (1,undefined) of (_,_) -> 42
42
```

```
ghci> case 1 :+: undefined of _ :+: _ -> 42
*** Exception: Prelude.undefined
```

```
ghci> case 1 :+: undefined of _ -> 42
42
```

BangPatterns extension

```
ghci> :set -XBangPatterns
ghci> foo !x = True
ghci> foo undefined
*** Exception: Prelude.undefined
```

In NAÏVE laziness implementation
Haskell is different



Repeated computations

```
f x = ((1+2) + x, 4 * x)
```

```
f (5+6)
→ ((1+2)+(5+6), 4*(5+6))
→ (3 + (5+6), 4 * (5+6))
→ (3+11, 4*(5+6))
→ (14, 4*(5+6))
→ (14, 4*11)
→ (14, 44)
```

Let

```
f y = let x = y in ((1+2)+x, 4*x)
```

```
f (5+6)
→ let x = (5+6) in ((1+2)+x, 4*x)
→ let x = (5+6) in (3+x, 4*x)
→ let x = 11 in (3+x, 4*x)
→ let x = 11 in (3+11, 4*x)
→ let x = 11 in (14, 4*x)
→ let x = 11 in (14, 4*11)
→ let x = 11 in (14, 44)
→ (11, 44)
```

Lazy patterns

Lazy pattern-matching

```
let (a,b) = p in ...
f ~(a,b) = ...
case p of ~(a,b) -> ...
(\ ~(a,b) -> ... )
```

Example

```
(**) :: (a->c) -> (b->d) -> (a,b) -> (c,d)
(**) f g ~(x,y) = (f x, g y)
```

```
ghci> :info const
const :: a -> b -> a
ghci> (const 1 ** const 2) undefined
(1,2)
```

```
f ~(a,b) = g a b
=>
f p = g (fst p) (snd p)
```

Example

Strict



h (a,b) = g a b

vs

Lazy



f ~(a,b) = g a b

```
g _ _ = 42
ghci> f undefined
42
ghci> h undefined
*** Exception: Prelude.undefined
ghci> h (undefined, undefined)
42
```

Example: splitAt

```
splitAt :: Int -> [a] -> ([a], [a])
splitAt n xs =
  if n <= 0
  then ([], xs)
  else case xs of
    [] -> ([], [])
    y:ys ->
      case splitAt (n-1) ys of
        (prefix, suffix) -> (y : prefix, suffix)
```

```
ghci> take 1000 . fst . splitAt 1000000 $ [1..]
```

> Lazy version produces output **much** faster

Simpliest HOF

```
twice :: (a -> a) -> a -> a
twice f x = f (f x)
```

why are HOF useful?

- > Common programming idioms
- > DSLs
- > Reasoning about programs

Some useful HOFs: map

```
map :: (a -> b) -> [a] -> [b]
ghci> map (+1) [1,3,5,7]
[2,4,6,8]
```

```
map f xs = [f x | x <- xs]
```

```
map f [] = []
```

```
map f (x:xs) = f x : map f xs
```

Some useful HOFs: filter

```
filter :: (a -> Bool) -> [a] -> [a]
```

```
ghci> filter even [1..10]
[2,4,6,8,10]
```

```
filter p xs = [x | x <- xs, p x]
```

```
filter p [] = []
```

```
filter p (x:xs)
```

```
  | p x          = x : filter p xs
```

```
  | otherwise    = filter p xs
```

Common Pattern on Lists

Usual pattern on lists (primitive recursion)

```
f [] = v  
f (x:xs) = x ⊕ f xs
```

Examples

```
sum [] = 0  
sum (x:xs) = x + sum xs  
  
product [] = 1  
product (x:xs) = x * product xs  
  
and [] = True  
and (x:xs) = x && and xs
```

foldr

```
sum = foldr (+) 0  
product = foldr (*) 1  
or = foldr (||) False  
and = foldr (&&) True  
  
foldr :: (a -> b -> b) -> b -> [a] -> b  
foldr f v [] = v  
foldr f v (x:xs) = f x (foldr f v xs)
```

Thinking foldr non-recursively

- > replace (:) with function
- > replace value with []

```
sum [1,2,3] = foldr (+) 0 [1,2,3]
            = 1+(2+(3+0))
product [1,2,3]
        = foldr (*) 1 [1,2,3]
        = foldr (*) 1 (1:2:3:[])
        = 1*(2*(3*1))
```

Example: length

```
length :: [a] -> Int
length [] = 0
length (_:xs) = 1 + length xs

length = foldr (\ _ n -> 1+n) 0
```

reverse and ++

```
reverse [] = []
reverse (x:xs) = reverse xs ++ [x]

reverse =
  foldr (\ x xs -> xs ++ [x]) []
```

```
(++ ys) = foldr (:) ys
```


Summary: foldr

- > Simple
- > Proving properties
- > Advanced optimizations

Declarative!

```
(.) :: (b->c) -> (a->b) -> (a->c)  
f . g = \ x -> f (g x)
```

```
odd :: Int -> Bool  
odd = not . even
```

- > **WHAT** to compute instead of **HOW** to compute it

```
sum . map (^2) . filter even
```

Other Useful HOFs on Lists

```
all :: (a -> Bool) -> [a] -> Bool
all p xs = and [p x | x <- xs]
```

```
any :: (a -> Bool) -> [a] -> Bool
any p xs = or [p x | x <- xs]
```

```
takeWhile :: (a->Bool) ->[a]->[a]
takeWhile p [] = []
takeWhile p (x:xs)
  | p x = x : takeWhile p ps
  | otherwise = []
```

```
dropWhile :: (a->Bool)->[a]->[a]
dropWhile p [] = []
dropWhile p (x:xs)
  | p x = dropWhile p ps
  | otherwise = x:xs
```

```
foldl
```

```
mapAccumL
mapAccumR
```

```
zip / unzip
zipWith / unzipWith
```

Some Summary: Haskell

Pros

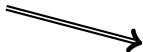
- ✓ Declarative
- ✓ Compiles \Rightarrow works
- ✓ Program reasoning

Cons

- ✗ Difficult to reason about efficiency
- ✗ Limited tool support for developers
- ✗ Requires ability to think abstractly:
think then type code

Tail Recursion

NB: Haskell's runtime is **different**
But the **reason** of stack overflow is the **same**



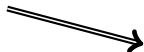
Usual recursion

```
make :: a -> Int -> [a]
make x n = if n < 1 then []
           else x : make x (n-1)
```

...	...
main	...
...	...
make 1 3	x=1 n=3 ...

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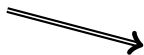
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main	...
...	...
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make 1 2	x=1 n=1 ...
make 1 1	x=1 n=1 ...

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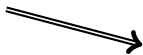
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make :: a -> Int -> [a]
make x n = if n < 1 then []
           else x : make x (n-1)
```

...	...
main	...
...	...
make 1 3	x=1 n=3 ...
make 1 2	x=1 n=1 ...
make 1 1	x=1 n=1 ...
make 1 0	x=1 n=0 ...

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Usual recursion

```
make :: a -> Int -> [a]
make x n = if n < 1 then []
           else x : make x (n-1)
```

...	...
main	...
...	...
make 1 3	x=1 n=3 ...
make 1 2	x=1 n=1 ...
make 1 1	x=1 n=1 ...
ret	[]

Tail Recursion

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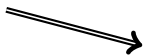
Usual recursion

```
make :: a -> Int -> [a]
make x n = if n < 1 then []
           else x : make x (n-1)
```

...	...
main	...
...	...
make 1 3	x=1 n=3 ...
make 1 2	x=1 n=1 ...
ret	[1]

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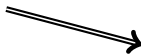
Usual recursion

```
make :: a -> Int -> [a]
make x n = if n < 1 then []
           else x : make x (n-1)
```

...	...
main	...
...	...
make 1 3	x=1 n=3 ...
ret	[1,1]

Tail Recursion

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Usual recursion

```
make :: a -> Int -> [a]
make x n = if n < 1 then []
          else x : make x (n-1)
```

...	...
main	...
...	...
ret	[1,1,1]

Tail Recursion


NB: Haskell's runtime is **different**
But the **reason** of stack overflow is the **same**

Usual recursion

```
make :: a -> Int -> [a]
make x n = if n < 1 then []
           else x : make x (n-1)
```

Tail recursion

```
make2 : a -> Int -> [a]
make2 x n = helper [] n
  helper acc n =
    if n < 1 then acc
    else helper (x F: acc) (n-1)
```



...	...
main	...
...	...
make2 1 3	x=1 n=3 ...

Tail Recursion


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make :: a -> Int -> [a]
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make2 x n = helper [] n
  helper acc n =
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    else helper (x F: acc) (n-1)
```



...	...
main	...
...	...
make2 1 3	x=1 n=3 ...
helper [] 3	acc=[] n=3 ...

Tail Recursion


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Usual recursion

```
make :: a -> Int -> [a]
make x n = if n < 1 then []
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make2 : a -> Int -> [a]
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  helper acc n =
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    else helper (x F: acc) (n-1)
```



...	...
main	...
...	...
make2 1 3	x=1 n=3 ...
helper [1] 2	acc=[1] n=2 ...

Tail Recursion


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Usual recursion

```
make :: a -> Int -> [a]
make x n = if n < 1 then []
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```

Tail recursion

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make2 : a -> Int -> [a]
make2 x n = helper [] n
  helper acc n =
    if n < 1 then acc
    else helper (x F: acc) (n-1)
```



...	...
main	...
...	...
make2 1 3	x=1 n=3 ...
helper [1,1] 1	acc=[1,1] n=1 ...

Usual recursion

```
make :: a -> Int -> [a]
make x n = if n < 1 then []
           else x : make x (n-1)
```

Tail recursion

```
make2 : a -> Int -> [a]
make2 x n = helper [] n
  helper acc n =
    if n < 1 then acc
    else helper (x F: acc) (n-1)
```

...	...
main	...
...	...
make2 1 3	x=1 n=3 ...
helper [1,1,1] 0	acc=[1,1,1] n=0 ...

Tail Recursion


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Usual recursion

```
make :: a -> Int -> [a]
make x n = if n < 1 then []
           else x : make x (n-1)
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Tail recursion

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make2 : a -> Int -> [a]
make2 x n = helper [] n
  helper acc n =
    if n < 1 then acc
    else helper (x F: acc) (n-1)
```



...	...
main	...
...	...
make2 1 3	x=1 n=3 ...
ret	[1,1,1]

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main	...
...	...
make2 1 3	x=1 n=3 ...
ret	[1,1,1]

Questions?