Functional Programming

ADT

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Before we start...

you can and **should** interrupt me and **ask** **any** **questions**

Functional programming

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Functional programming

a *style* of programming in which basic method of computation is function application

```
int counter = 0;
for (int i = 0; i < n; i++)
  counter += i;
```

variable assignment

```
sum [1..n]

fold (+) [1..n] 0
```

- function application
- declarative
- efficiency — compiler’s job

Functional programming

a *style* of programming in which basic method of computation is function application

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int counter = 0;
for (int i = 0; i < n; i++)
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```

- function application
- declarative
- efficiency — compiler's job

Math	Haskell
$f(x)$	`f x`
$f(x,y)$	`f x y`
$f(g(x))$	`f (g x)`
$f(x,g(x))$	`f x (g y)`
$f(x)g(y)$	`f x * g y`

Types

What is a *type*?

Types

What is a *type*?

a *collection* of its values

Example: `Bool = True "+" False`

Strongly Statically Typed

- Safer
- Faster since no type checking in runtime
- Specifies function behaviour!

Specifies function behaviour

- `Int -> Bool`
- `Int -> Int -> Int`

Basic types

- `Bool`
- `Int`,`Integer`
- `Float`,`Double`
- `f :: a -> a`
- `[a]`
- `Char, String = [Char]`
- `:type <Expr>`

Types and Primitive Data

Basic types

- `Bool`
- `Int`,`Integer`
- `Float`,`Double`
- `f :: a -> a`
- `[a]`
- `Char`, `String = [Char]`
- `:type <Expr>`

Tuples

```
(42,"Hello!") :: (Int,String)

(True,'r',12312.123123)
```

- min tuple size: 2
- max tuple size: ≥ 15
  (62 in ghc)

Basic types

- Bool
- Int, Integer
- Float, Double
- f :: a -> a
- [a]
- Char, String = [Char]
- :type <Expr>

Tuples

```
(42,"Hello!") :: (Int,String)

(True,'r',12312.123123)
```

- min tuple size: 2
- max tuple size: ≥ 15 (62 in ghc)

Lists

```
[]
[1,2,3,4,9]
1 : [] ≡ [1]
ghci> ['H', 'e', 'l', 'l', 'o']
"Hello"
ghci> "hello" ++ " " ++ "world"
"hello world"
ghci> let b = [[1,2,3,4],[3,4,5,6,7]]
ghci> b
[[1,2,3,4],[3,4,5,6,7]]
ghci> b ++ [[1,1,1]]
[[1,2,3,4],[3,4,5,6,7],[1,1,1]]
ghci> head [5,4,3,2,1]
5
ghci> tail [5,4,3,2,1]
[4,3,2,1]
ghci> head []
*** Exception: Prelude.head: empty list
```

Lists provides no info about size, tuples does!

Make your own types: Algebraic Data Types

- a-la enum + structures in C
- ADT = union + product + exponential types

Trivial union types: enums (0-arity)

```
--      [1]      [2] [3] [2][3] [2] [3] [2]
data Cardinal = North | East | South | West
-- [1]: Type constructor
-- [2]: Data constructor
-- [3]: The pipe operator that separates data constructors
```

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data Cardinal = North | East | South | West
-- [1]: Type constructor
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```

Pattern Matching

```
hasPole :: Cardinal -> Bool
hasPole x =
  if (x == North) || (x == South)
  then True
  else False
```

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- ADT = union + product + exponential types

Trivial union types: enums (0-arity)

```haskell
--       [1]        [2]  [3]  [2][3]  [2]  [3]  [2]
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Pattern Matching

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hasPole x =
  if (x == North) || (x == South)
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```haskell
hasPole North = True
hasPole South = True
hasPole _     = False
```

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hasPole :: Cardinal -> Bool
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  then True
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```

```haskell
hasPole North = True
hasPole South = True
hasPole _     = False

hasPole x = case x of
  North -> True
  South -> True
  _     -> False
```

Make your own types: Algebraic Data Types

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Trivial union types: enums (0-arity)

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-- [1]: Type constructor
-- [2]: Data constructor
-- [3]: The pipe operator that separates data constructors
```

Pattern Matching

```haskell
hasPole :: Cardinal -> Bool
hasPole x =
  if (x == North) || (x == South)
  then True
  else False
```

```haskell
hasPole x = x `elem` [South, North]
```

```haskell
hasPole North = True
hasPole South = True
hasPole _     = False

hasPole x = case x of
  North -> True
  South -> True
  _     -> False
```

Make your own types: Algebraic Data Types

- a-la enum + structures in C
- ADT = union + product + exponential types

Trivial union types: enums (0-arity)

```
--       [1]       [2] [3] [2][3] [2] [3] [2]
data Cardinal = North | East | South | West deriving Eq
-- [1]: Type constructor
-- [2]: Data constructor
-- [3]: The pipe operator that separates data constructors
```

Pattern Matching

```
hasPole :: Cardinal -> Bool
hasPole x =
  if (x == North) || (x == South)
  then True
  else False
```

```
hasPole x = x `elem` [South, North]
```

↑ **Instance of Eq**

```
hasPole North = True
hasPole South = True
hasPole _     = False

hasPole x = case x of
  North -> True
  South -> True
  _     -> False
```

Simple Product Type

```haskell
--    [1]      [2]          [3]
data Point = Point Double Double
-- [1]: Type constructor.
-- [2]: Data constructor.
-- [3]: Types wrapped.
```

Simple Product Type

```
--     [1]     [2]        [3]
data Point = Point Double Double
-- [1]: Type constructor.
-- [2]: Data constructor.
-- [3]: Types wrapped.
ghci> :type Point
Point :: Double -> Double -> Point
ghci> a = Point 3 4
ghci> a
Point 3.0 4.0
ghci> a = Point 3 4
ghci> b = Point 1 2
```

Simple Product Type

```
--     [1]      [2]        [3]
data Point = Point Double Double
-- [1]: Type constructor.
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ghci> :type Point
Point :: Double -> Double -> Point
ghci> a = Point 3 4
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Point 3.0 4.0
ghci> a = Point 3 4
ghci> b = Point 1 2

dist (Point x1 y1) (Point x2 y2) =
  sqrt ((x1 - x2)^2 + (y1 - y2)^2)

ghci> dist a b
2.8284271247461903
```

Product types

Simple Product Type

```
--    [1]      [2]         [3]
data Point = Point Double Double
-- [1]: Type constructor.
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  sqrt ((x1 - x2)^2 + (y1 - y2)^2)

ghci> dist a b
2.8284271247461903
```

Polymorphic Data Types

```
data PPoint a = PPoint a a
distP (PPoint x1 y1) (PPoint x2 y2) =
  sqrt ((x1 - x2)^2 + (y1 - y2)^2)
```

Product types

Simple Product Type

```haskell
--     [1]      [2]         [3]
data Point = Point Double Double
-- [1]: Type constructor.
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ghci> :type Point
Point :: Double -> Double -> Point
ghci> a = Point 3 4
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Point 3.0 4.0
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dist (Point x1 y1) (Point x2 y2) =
  sqrt ((x1 - x2)^2 + (y1 - y2)^2)

ghci> dist a b
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```

Polymorphic Data Types

```haskell
data PPoint a = PPoint a a
distP (PPoint x1 y1) (PPoint x2 y2) =
  sqrt ((x1 - x2)^2 + (y1 - y2)^2)
```

```haskell
ghci> :kind Point
Point :: *
ghci> :kind PPoint
PPoint :: * -> *
ghci> :info (,)
type (,) :: * -> * -> *
data (,) a b = (,) a b
ghci> :t distP
distP :: Floating a => PPoint a
  -> PPoint a -> a
```

Union Types

```haskell
data Point = Point2D Double Double | Point3D Double Double Double

pointToList :: Point -> [Double]
pointToList (Point2D x y) = [x, y]
pointToList (Point3D x y z) = [x, y, z]
*Main> a = Point2D 3 4
*Main> b = Point3D 3 4 5
*Main> pointToList a
[3.0,4.0]
*Main> pointToList b
[3.0,4.0,5.0]
```

	Product types	Sum types
Example	`data (,) a b = (,) a b`	`data Bool = False | True`
Intuition	a **and** b	a **or** b

Built-in types *behaves* like ADTs

```
data Char = '\NUL' | ... | 'a'
  | 'b' | 'c' | 'd' | ...
  | '\1114111'
data Int = -9223372036854775808 |...
  | -2 | -1 | 0 | 1 | 2 | ...
  | 9223372036854775807
data Integer = ... | -2 | -1 | 0
  | 1 | 2 | ...

isAnswer :: Integer -> Bool
isAnswer 42 = True
isAnswer _  = False
```

More on Pattern Matching

Built-in types ***behaves*** like ADTs

```
data Char = '\NUL' | ... | 'a'
  | 'b' | 'c' | 'd' | ...
  | '\1114111'
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  | 9223372036854775807
data Integer = ... | -2 | -1 | 0
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isAnswer :: Integer -> Bool
isAnswer 42 = True
isAnswer _  = False
```

Pattern Matching Semantics

```
bar 1 2 = 3
bar 0 _ = 5

bar 0 7
bar 2 1
bar 1 (5-3)
bar 1 undefined
bar 0 undefined
```

More on Pattern Matching

Built-in types *behaves* like ADTs

```
data Char = '\NUL' | ... | 'a'
  | 'b' | 'c' | 'd' | ...
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isAnswer :: Integer -> Bool
isAnswer 42 = True
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Pattern Matching Semantics

```
bar 1 2 = 3
bar 0 _ = 5

bar 0 7 -- fail, success
bar 2 1 -- fail, fail
bar 1 (5-3) -- success
bar 1 undefined -- diverge
bar 0 undefined -- success
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More on Pattern Matching

Built-in types *behaves* like ADTs

```
data Char = '\NUL' | ... | 'a'
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data Int = -9223372036854775808 |...
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Pattern Matching Semantics

```
bar 1 2 = 3
bar 0 _ = 5

bar 0 7 -- fail, success
bar 2 1 -- fail, fail
bar 1 (5-3) -- success
bar 1 undefined -- diverge
bar 0 undefined -- success
```

Exhaustive?

```
repl :: String -> String
repl " " = " "
repl (x:xs) = x:x:repl xs
ghci> repl "a"
"aa*** Exception: ...
Non-exhaustive patterns ...

-fwarn-incomplete-patterns (-W, -Wall)
```

Maybe

```
ghci> :info Maybe
type Maybe :: * -> *
data Maybe a = Nothing | Just a

ghci> head []
*** Exception: Prelude.head: empty
  list
```

Maybe

```
ghci> :info Maybe
type Maybe :: * -> *
data Maybe a = Nothing | Just a

ghci> head []
*** Exception: Prelude.head: empty
  list

safeHead :: [a] -> Maybe a
safeHead []     = Nothing
safeHead (x:_) = Just x

ghci> safeHead []
Nothing
ghci> safeHead [1, 2, 3]
Just 1
```

Standard ADTs

Maybe

```
ghci> :info Maybe
type Maybe :: * -> *
data Maybe a = Nothing | Just a

ghci> head []
*** Exception: Prelude.head: empty
  list

safeHead :: [a] -> Maybe a
safeHead []    = Nothing
safeHead (x:_) = Just x

ghci> safeHead []
Nothing
ghci> safeHead [1, 2, 3]
Just 1
```

Either

```
ghci> :info Either
type Either :: * -> * -> *
data Either a b = Left a | Right b

safeHead :: [a] -> Either String a
safeHead []    =
  Left "safeHead: empty list"
safeHead (x:_) = Right x

ghci> safeHead []
"safeHead: empty list"
ghci> safeHead [1, 2, 3]
Right 1
```

- **Distinctness**

$$\forall j \neq i . C_i(x) \neq C_j(y)$$

- Injectivity

$$C_{ij}(x_1, \ldots, x_{n_{ij}} = C_{ij}(y_1, \ldots, y_{n_{ij}}) \Rightarrow \forall k . x_k = y_k$$

- Exhaustiveness

$$x \text{ of some ADT } \Rightarrow \exists i, n . x = C_i(y_1, \ldots, y_n)$$

- Selection

$$\exists s_i^k : s_i^k(C_k(x_{k_1}, \ldots, x_{k_n})) = x_{k_i}$$

Distinctness

- Start from different constructors ⇒ different values

```
neqLists []     (_:_)   = True
neqLists (_:_)  []      = True
neqLists (x:xs) (y:ys) =
  (x /= y) || (neqLists xs ys)
neqLists []     []      = False
```

- Distinctness

$$\forall j \neq i. C_i(x) \neq C_j(y)$$

- **Injectivity**

$$C_{ij}(x_1, \ldots, x_{n_{ij}}) = C_{ij}(y_1, \ldots, y_{n_{ij}}) \Rightarrow \forall k. x_k = y_k$$

- Exhaustiveness

$$x \text{ of some ADT } \Rightarrow \exists i, n. x = C_i(y_1, \ldots, y_n)$$

- Selection

$$\exists s_i^k : s_i^k(C_k(x_{k_1}, \ldots, x_{k_n})) = x_{k_i}$$

Injectivity

- Same values ⇒ start from the same constructor, and arguments are equal pairwise

```
eqLists []       []       = True
eqLists []       (_:_)    = False
eqLists (_:_)    []       = False
eqLists (x:xs) (y:ys) =
  (x == y) && eqLists xs ys
```

- Distinctness

$$\forall j \neq i. C_i(x) \neq C_j(y)$$

- Injectivity

$$C_{ij}(x_1, \ldots, x_{n_{ij}}) = C_{ij}(y_1, \ldots, y_{n_{ij}}) \Rightarrow \forall k. x_k = y_k$$

- **Exhaustiveness**

$$x \text{ of some ADT } \Rightarrow \exists i, n. x = C_i(y_1, \ldots, y_n)$$

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$$\exists s_i^k : s_i^k(C_k(x_{k_1}, \ldots, x_{k_n})) = x_{k_i}$$

Exhaustiveness

- Values of some ADT starts from one of constructors listed in the type definition only

```
evenLength [] = True
evenLength (_:_:tl) = evenLength tl

ghci>:set -fwarn-incomplete-patterns
...: warning: [-Wincomplete-patterns]
  Pattern match(es) are non-exhaustive
  In an equation for ‘evenLength’:
    Patterns not matched: [_]
 |
 | evenLength [] = True
 | ^^^^^^^^^^^^^^^^^^^^...
```

Formal ADT Axioms

- Distinctness

$$\forall j \neq i. C_i(x) \neq C_j(y)$$

- Injectivity

$$C_{ij}(x_1, \ldots, x_{n_{ij}}) = C_{ij}(y_1, \ldots, y_{n_{ij}}) \Rightarrow \forall k. x_k = y_k$$

- Exhaustiveness

$$x \text{ of some ADT } \Rightarrow \exists i, n. x = C_i(y_1, \ldots, y_n)$$

- **Selection**

$$\exists s_i^k : s_i^k(C_k(x_{k_1}, \ldots, x_{k_n})) = x_{k_i}$$

Selection

- One can select an (sub)-element via pattern-matching

```
somefunct [] =
  -- no argument of empty list
  ...
somefunct (h:tl) =
 ... h ... tl ... h ...
```

ADT definition can be recursive

```haskell
data List a = Nil | Cons a (List a)
data List a = []  | (:)  a (List a)

Nil :: [a]
Cons 1 Nil :: [Int]
Cons 2 (Cons 1 Nil) :: [Int]
```

type and newtype

```haskell
type FirstName1 = String

newtype FirstName2 = FirstName2 String
```

Recursive types

ADT definition can be recursive

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data List a = Nil | Cons a (List a)
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Nil :: [a]
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Have **same** constructors

New constructor

type and newtype

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Recursive types

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```

Have **same** constructors

New constructor

type and newtype

```
type FirstName1 = String

newtype FirstName2 = FirstName2 String
```

Example

```
unF1 :: FirstName1 -> String
unF1 = id

unF2 :: FirstName2 -> String
unF2 (FirstName2 s) = s

ghci> unF1 "a"
"a"
ghci> unF1 (FirstName2 "a")
"a"
```

Recursive types

ADT definition can be recursive

```
data List a = Nil | Cons a (List a)
data List a = []  | (:)  a (List a)

Nil :: [a]
Cons 1 Nil :: [Int]
Cons 2 (Cons 1 Nil) :: [Int]
```

Have **same** constructors

New constructor

type and newtype

```
type FirstName1 = String

newtype FirstName2 = FirstName2 String
```

Example

```
unF1 :: FirstName1 -> String
unF1 = id

unF2 :: FirstName2 -> String
unF2 (FirstName2 s) = s

ghci> unF1 "a"
"a"
ghci> unF1 (FirstName2 "a")
"a"
```

newtype

- Type safety
- Exactly one constructor and one field
- Can't be recursive

Exponential types (functions)

```
data Endom a = Endom (a -> a)
appEndom :: Endom a -> a -> a
appEndom (Endom f) = f

ghci> e = Endom (\n -> 2 * n + 1)
ghci> :t e
e :: Num a => Endom a
```

- $|a \to b| = |a|^{|b|}$
- Functions — first class values

Why it is called Algebraic?

Data.Void	**Void**	0
`()`	`()`	1
Bool	`data Bool = False | True`	1+1
Maybe	`data Maybe a = Nothing | Just a`	1+|a|
Either	`data Either a b = Left a | Right b`	|a|+|b|
Tuple	`(a, b)`	|a| * |b|
Function	`a -> b`	$\|b\|^{\|a\|}$
2D or 3D point	`data Point a = Point2D a a | Point3D a a a`	$\|a\|^2 + \|a\|^3$

Example: Arithmetic Expressions

Variant 1

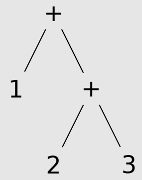
```
data Expr =
   | Const Int
   | VarCalledX
   | Plus     Expr Expr
   | Asterisk Expr Expr
   | Dash     Expr Expr
   | Slash    Expr Expr
```

Variant 2

```
data Op = Plus | Asterisk
   | Dash | Slash
data Expr =
  | Const Int
  | VarCalledX
  | BinOp Op Expr Expr
```

Looks exactly like AST

```
Plus (Const 1,
  Plus (Const 2,
        Const 3))
```



ADT: What can go Wrong?

should be **safe by construction**

```haskell
data Expr = Plus Expr Expr | Const Int | ...
x = Plus (Const 1, Plus (Const 2, Const 3))
                -- vs
data Expr = Plus [Expr] | Const Int | ...
y = Plus [ Const 1, Const 2, Const 3 ]
```

ADT: What can go Wrong?

should be **safe by construction**

```
data Expr = Plus Expr Expr | Const Int | ...
x = Plus (Const 1, Plus (Const 2, Const 3))
               -- vs
data Expr = Plus [Expr] | Const Int | ...
y = Plus [ Const 1, Const 2, Const 3 ]
z = Plus [ ]  -- semantics?
```

ADT: What can go Wrong?

should be safe by construction

```
data Expr = Plus Expr Expr | Const Int | ...
x = Plus (Const 1, Plus (Const 2, Const 3))
                -- vs
data Expr = Plus [Expr] | Const Int | ...
y = Plus [ Const 1, Const 2, Const 3 ]
z = Plus [ ]  -- semantics?
```

data should have a unique representation

```
data Expr =
  | Plus Expr Expr
  | Oper Op [Expr]
  | ...
```

Usual Union Data Definition

```
data SimplePerson = SimplePerson String String String Int String
firstPerson = SimplePerson "Alan" "Smith" "asmith@gmail.com" 42 "Lawyer"
incomplete  = SimplePerson "Michael" "Smith" "msmith@gmail.com" 42
complete    = incomplete "Dancer"
```

Usual Union Data Definition

```haskell
data SimplePerson = SimplePerson String String String Int String
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complete    = incomplete "Dancer"
```

Records

- are an extension of union ADT that allow fields to be named:

```haskell
data Person = Person
  { age :: Int
  , name :: String
  }
ghci> a = Person 3 "a"
ghci> a
Person {age = 3, name = "a"}
ghci> age a
3
ghci> b = a {name = "BB"} --"update"
ghci> b
Person {age = 3, name = "BB"}
```

```haskell
ghci> grouwUp person =
  person {age = age person + 1}
ghci> c = growUp a
ghci> c
Person {age = 4, name = "a"}
```

Usual Union Data Definition

```haskell
data SimplePerson = SimplePerson String String String Int String
firstPerson = SimplePerson "Alan" "Smith" "asmith@gmail.com" 42 "Lawyer"
incomplete  = SimplePerson "Michael" "Smith" "msmith@gmail.com" 42
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Records

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data Person = Person
  { age :: Int
  , name :: String
  }
ghci> a = Person 3 "a"
ghci> a
Person {age = 3, name = "a"}
ghci> age a
3
ghci> b = a {name = "BB"} --"update"
ghci> b
Person {age = 3, name = "BB"}
```

```haskell
ghci> grouwUp person =
  person {age = age person + 1}
ghci> c = growUp a
ghci> c
Person {age = 4, name = "a"}
```

RecordWildCards

```haskell
lowerCaseName :: Person -> String
lowerCaseName (Person { name }) =
  map toLower name
```

Records

Usual Union Data Definition

```haskell
data SimplePerson = SimplePerson String String String Int String
firstPerson = SimplePerson "Alan" "Smith" "asmith@gmail.com" 42 "Lawyer"
incomplete  = SimplePerson "Michael" "Smith" "msmith@gmail.com" 42
complete    = incomplete "Dancer"
```

Records

- are an extension of union ADT that allow fields to be named:

```haskell
data Person = Person
  { age :: Int
  , name :: String
  }
ghci> a = Person 3 "a"
ghci> a
Person {age = 3, name = "a"}
ghci> age a
3
ghci> b = a {name = "BB"} --"update"
ghci> b
Person {age = 3, name = "BB"}
```

```haskell
ghci> grouwUp person =
  person {age = age person + 1}
ghci> c = growUp a
ghci> c
Person {age = 4, name = "a"}
```

RecordWildCards

```haskell
lowerCaseName :: Person -> String
lowerCaseName (Person { name }) =
  map toLower name
lowerCaseName (Person { .. }) =
  map toLower name
f (Person {age = 3, ..})= name++"b"
```

More on Records: common field labels

common field labels

```
data Point a = Point2D {xCord :: a, yCord :: a}
| Point3D {xCord :: a, yCord :: a, zCord :: a}
ghci> p1 = Point2D 1.0 1.0
ghci> p2 = Point3D 2.0 2.0 2.0
ghci> xCord p1
1.0
ghci> xCord p2
2.0
```

- Field labels has global scope
- Thus, common labels may be wihtin one data type only

```
data Point1D a = Point1D {xCord :: a}
ghci>
Error: Multiple declarations of 'xCord'.
```

Example: Network package

```haskell
data ConnectionState =
  Connecting | Connected | Disconnected
data ConnectionInfo = ConnectionInfo
  { state :: ConnectionState,
    server :: InetAddr,
    last_ping_time :: Maybe Time,
    last_ping_id :: Maybe Int,
    session_id :: Maybe String,
    when_initiated :: Maybe Time,
    when_disconnected :: Maybe Time
  }
```

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  }
```

```haskell
-- better
data ConnectingD = ConnectingD
  { whenInitiated :: Time }
data ConnectedD = ConnectedD
  { lastPing  :: Maybe (Time, Int),
    sessionId :: String }
data DisconnectedD =
  DisconnectedD {
    whenDisconnected :: Time }
```

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  DisconnectedD {
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```

```haskell
-- but ugly
data ConnectionState =
  Connecting ConnectingD
  | Connected ConnectedD
  | Disconnected DisconnectedD
data ConnectionInfo = ConnectionInfo
  { state  :: ConnectionState,
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    when_initiated :: Maybe Time,
    when_disconnected :: Maybe Time
  }
```

```haskell
-- +/- good
data ConnectionState =
  Connecting  { whenInitiated :: Time }
  | Connected { lastPing ::Maybe(Time,Int),
                sessionId::String }
  | Disconnected { whenDisconnected::Time }
data ConnectionInfo = ConnectionInfo
  { state  :: ConnectionState,
    server :: InetAddr
  }
```

```haskell
-- better
data ConnectingD = ConnectingD
  { whenInitiated :: Time }
data ConnectedD = ConnectedD
  { lastPing  :: Maybe (Time, Int),
    sessionId :: String }
data DisconnectedD =
  DisconnectedD {
    whenDisconnected :: Time }
```

```haskell
-- but ugly
data ConnectionState =
  Connecting ConnectingD
  | Connected ConnectedD
  | Disconnected DisconnectedD
data ConnectionInfo = ConnectionInfo
  { state  :: ConnectionState,
    server :: InetAddr
  }
```

Pure function

Pure Functions

Pure function

- Deterministic
  - No use of global mutable data
  - No dependence on date, time, rand and so on
- No side effects
  - No input/output
  - Purely or impurely call impure functions

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Remarks

- Is a property of a *function*, not a language
- Easier to optimize: e.g. inlining
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Mathematical function **vs** pure function

- Non-termination
- Crash

	Math	Programming
Always returns a result	function	total function
May not return a result	partial function	function

Datatype Cardinality

A number of *different* terms (values) that populate the type.

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Guess type cardinality (pure functions)

```
?> () -> Int
?> Int -> ()
?> Void -> Int
```

```
?> Int -> Void
?> a -> ()
?> Void -> a
```

Guess

What these *pure* functions can do? What about cardinality?

```haskell
? a -> a
? [a] -> [a]
? [a] -> Bool
? (a -> b) -> [a] -> [b]
? (a -> a -> Bool) -> [a] -> [a]
? (a -> a -> Ordering) -> [a] -> [a] where data Ordering = LT | EQ | GT
? [a] -> [b] -> [(a,b)]
? Maybe a -> Maybe b -> (a -> b -> c) -> Maybe c
? Int -> a -> [a]
? [a] -> Int -> a
```

Guess

What these *pure* functions can do? What about cardinality?

```
? a -> a                                                      id x = x
? [a] -> [a]
? [a] -> Bool
```

```
isEmpty [] = True
isEmpty _  = False
```

```
? (a -> b) -> [a] -> [b]
```

```
map _ []     = []
map f (x:xs) = f x : map f xs
```

```
? (a -> a -> Bool) -> [a] -> [a]                              filter
? (a -> a -> Ordering) -> [a] -> [a] where data Ordering = LT | EQ | GT
? [a] -> [b] -> [(a,b)]                                       zip
? Maybe a -> Maybe b -> (a -> b -> c) -> Maybe c
? Int -> a -> [a]
```

```
replicate 0 _ = []
replicate n x = x : replicate (n-1) x
```

```
? [a] -> Int -> a
```

```
(x:_ ) !! 0 = x
(_:xs) !! n = xs !! (n-1)
```

again: **type specifies behaviour**

Questions?