

## Zippers and Lenses

**Daniil Berezun**

**danya.berezun@gmail.com**

2022

# Outline for section 1

- 1 Zippers
  - Motivation
  - List Zipper
  - How to derive a zipper?
  - Tree Zipper

- 2 “Optics”
  - Lens: Motivation
  - **Control.Lens**
  - More on **Control.Lens**
  - Prism
  - Traversal

› Functional data structures are immutable  $\Rightarrow$  *hard and expensive to modify*

```
data List a = Nil | Cons a (List a)
```

```
update :: List a -> Int -> a -> List a
```

```
update Nil _ _ = Nil
```

```
update (Cons _ xs) n _ = Cons a xs
```

```
update (Cons x xs) n a = Cons x $ update a xs
```

- Functional data structures are immutable  $\Rightarrow$  *hard and expensive to modify*

```
data List a = Nil | Cons a (List a)
```

```
update :: List a -> Int -> a -> List a
```

```
update Nil _ _ = Nil
```

```
update (Cons _ xs) n _ = Cons a xs
```

```
update (Cons x xs) n a = Cons x $ update a xs
```

- Let's construct another data structure s.t.:
- represents the original data structure
  - has an ability to *navigate through* the structure focusing on some sub-structure
  - *allows efficient modification* of the element in focus (aka *hole*)

# Immutability and Modification

› Functional data structures are immutable  $\Rightarrow$  *hard* and *expensive to modify*

```
data List a = Nil | Cons a (List a)
```

```
update :: List a -> Int -> a -> List a
```

```
update Nil _ _ = Nil
```

```
update (Cons _ xs) n _ = Cons a xs
```

```
update (Cons x xs) n a = Cons x $ update a xs
```

› Let's construct another data structure s.t.:

- represents the original data structure
- has an ability to *navigate through* the structure focusing on some sub-structure
- *allows efficient modification* of the element in focus (aka *hole*)



# Immutability and Modification

› Functional data structures are immutable  $\Rightarrow$  *hard* and *expensive to modify*

```
data List a = Nil | Cons a (List a)
```

```
update :: List a -> Int -> a -> List a
```

```
update Nil _ _ = Nil
```

```
update (Cons _ xs) n _ = Cons a xs
```

```
update (Cons x xs) n a = Cons x $ update a xs
```

› Let's construct another data structure s.t.:

- represents the original data structure
- has an ability to *navigate through* the structure focusing on some sub-structure
- *allows efficient modification* of the element in focus (aka *hole*)



# Immutability and Modification

› Functional data structures are immutable  $\Rightarrow$  *hard* and *expensive to modify*

```
data List a = Nil | Cons a (List a)
```

```
update :: List a -> Int -> a -> List a
```

```
update Nil _ _ = Nil
```

```
update (Cons _ xs) n _ = Cons a xs
```

```
update (Cons x xs) n a = Cons x $ update a xs
```

› Let's construct another data structure s.t.:

- represents the original data structure
- has an ability to *navigate through* the structure focusing on some sub-structure
- *allows efficient modification* of the element in focus (aka *hole*)



# Immutability and Modification

› Functional data structures are immutable  $\Rightarrow$  *hard* and *expensive to modify*

```
data List a = Nil | Cons a (List a)
```

```
update :: List a -> Int -> a -> List a
```

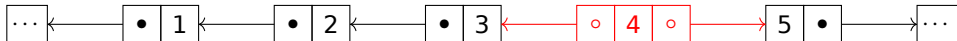
```
update Nil _ _ = Nil
```

```
update (Cons _ xs) n _ = Cons a xs
```

```
update (Cons x xs) n a = Cons x $ update a xs
```

› Let's construct another data structure s.t.:

- represents the original data structure
- has an ability to *navigate through* the structure focusing on some sub-structure
- *allows efficient modification* of the element in focus (aka *hole*)





# Immutability and Modification

- Functional data structures are immutable  $\Rightarrow$  *hard* and *expensive to modify*

```
data List a = Nil | Cons a (List a)
```

```
update :: List a -> Int -> a -> List a
```

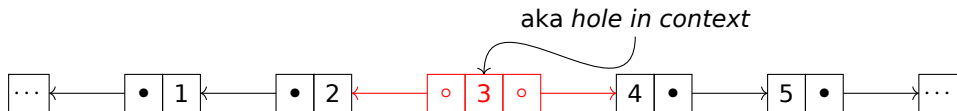
```
update Nil _ _ = Nil
```

```
update (Cons _ xs) n _ = Cons a xs
```

```
update (Cons x xs) n a = Cons x $ update a xs
```

- Let's construct another data structure s.t.:

- represents the original data structure
- has an ability to *navigate through* the structure focusing on some sub-structure
- *allows efficient modification* of the element in focus (aka *hole*)



## List Zipper: traversing

```
type ListZipper a = (a, ContextLZ a)
type ContextLZ a = ([a],[a])
-- construct list zipper
makeLZ :: [a] -> ListZipper a
makeLZ (x:xs) = (x, ([],xs))
-- move focus forward
forwardLZ :: ListZipper a -> ListZipper a
forwardLZ (e, (xs, y:ys)) = (y, (e:xs, ys))
-- move focus back
backwardLZ :: ListZipper a -> ListZipper a
backwardLZ (a, (x:xs, ys)) = (x, (xs, a:ys))
-- extract list from list zipper
fromLZ :: ListZipper a -> [a]
fromLZ (x, ([], xs)) = x:xs
fromLZ z = fromLZ . backwardLZ $ z
```

```
-- usage examples
ghci> lz = makeLZ [0..3]
(0, ([], [1,2,3]))
ghci> forward lz
(1, ([0], [2,3]))
ghci> let lz' =
      (forward . forward) lz
(2, ([1,0], [3]))
ghci> backward lz'
(1, ([0], [2,3]))
ghci> fromLZ lz'
[0,1,2,3]
```

```

-- update element in hole
updateLZ :: a -> ListZipper a -> ListZipper a
updateLZ a (_, ctx) = (a, ctx)
-- insert element in hole
insertLZ :: a -> ListZipper a -> ListZipper a
insertLZ a (b, (xs, ys)) = (a, (xs, b:ys))
-- remove element in focus from list
removeLZ :: ListZipper a -> ListZipper a
removeLZ (_, (x:xs, [])) = (x, (xs, []))
removeLZ (_, (xs, y:ys)) = (y, (xs, ys))
-- usage examples:
ghci> fromLZz . updateLZ 22 . fowardLZ . fowardLZ . makeLZ $ [0..3]
[0,1,22,3]
ghci> fromLZ . insertLZ 11 . insertLZ 10 . forward . forward . makeLZ $ [0..3]
[0,1,11,10,2,3]

```

# Remember Types Algebra?

## Tuples

**type Triple** a = (a, (a, a))

**type PairPair** a = ((a, a), (a, a))

## How many elements of type?

**Triple** =  $A * (A * A)$  =  $A^3$

**PairPair** =  $(A * A)^2$  =  $A^4$

# Remember Types Algebra?

## Tuples

```
type Triple a = (a, (a,a))
type PairPair a = ((a,a), (a,a))
```

## How many elements of type?

```
Triple = A * (A * A) = A3
PairPair = (A * A)2 = A4
```

## Zippers

```
type TripleZ a = (a, CntxTZ a)
data CntxTZ a = CTZ1 a a | CTZ2 a a | CTZ3 a a

type PairPairZ a = (a, CntxPPZ a)
data CntxPPZ a = CPPZ1 a a a | CPPZ2 a a a | CPPZ3 a a a | CPPZ4 a a a
```

# Remember Types Algebra?

## Tuples

```
type Triple a = (a, (a, a))
type PairPair a = ((a, a), (a, a))
```

## How many elements of type?

```
Triple = A * (A * A) = A3
PairPair = (A * A)2 = A4
```

## Zippers

```
type TripleZ a = (a, CntxTZ a)
data CntxTZ a = CTZ1 a a | CTZ2 a a | CTZ3 a a

type PairPairZ a = (a, CntxPPZ a)
data CntxPPZ a = CPPZ1 a a a | CPPZ2 a a a | CPPZ3 a a a | CPPZ4 a a a
```

## In term of type theory algebra

```
TripleZ (X) = X3
CntxTZ (X) = X2 + X2 + X2 = 3 * X2

PairPairZ (X) = X4
CntxPPZ (X) = X3 + X3 + X3 + X3 = 4 * X3
```

# Remember Types Algebra?

## Tuples

```
type Triple a = (a, (a, a))
type PairPair a = ((a, a), (a, a))
```

## How many elements of type?

```
Triple = A * (A * A) = A3
PairPair = (A * A)2 = A4
```

## Zippers

```
type TripleZ a = (a, CntxTZ a)
data CntxTZ a = CTZ1 a a | CTZ2 a a | CTZ3 a a

type PairPairZ a = (a, CntxPPZ a)
data CntxPPZ a = CPPZ1 a a a | CPPZ2 a a a | CPPZ3 a a a | CPPZ4 a a a
```

## In term of type theory algebra

```
TripleZ (X) = X3
CntxTZ (X) = X2 + X2 + X2 = 3 * X2
PairPairZ (X) = X4
CntxPPZ (X) = X3 + X3 + X3 + X3 = 4 * X3
derivative!!!
```

## Lists

$$L(X) = 1 + X + X^2 + X^3 + \dots$$

$$L(X) = 1 + X * (1 + X + X^2 + X^3 + \dots)$$

$$L(X) = 1 + X * L(X)$$

## Further

$$L(X) - X * L(X) = 1$$

$$L(X) * (1 - X) = 1$$

$$L(X) = \frac{1}{1-X}$$



## Lists

$$\begin{aligned} L(X) &= 1 + X + X^2 + X^3 + \dots \\ L(X) &= 1 + X * (1 + X + X^2 + X^3 + \dots) \\ L(X) &= 1 + X * L(X) \end{aligned}$$

## Further

$$\begin{aligned} L(X) - X * L(X) &= 1 \\ L(X) * (1 - X) &= 1 \\ L(X) &= \frac{1}{1-X} \end{aligned}$$

## Derivate

$$\begin{aligned} L(X) &= \frac{1}{1-X} \\ L'(X) &= \frac{1}{(1-X)^2} \\ L'(X) &= L(X) * L(X) \end{aligned}$$

## Derivate: Alternative syntax

$$\begin{aligned} L &= 1 + X * L \\ \frac{\partial L}{\partial X} &= \frac{\partial}{\partial X} (1 + X * L) = L + X * \frac{\partial L}{\partial X} \\ \frac{\partial L}{\partial X} &= \frac{L}{1-X} = L^2 \end{aligned}$$

## Lists

$$\begin{aligned} L(X) &= 1 + X + X^2 + X^3 + \dots \\ L(X) &= 1 + X * (1 + X + X^2 + X^3 + \dots) \\ L(X) &= 1 + X * L(X) \end{aligned}$$

## Further

$$\begin{aligned} L(X) - X * L(X) &= 1 \\ L(X) * (1 - X) &= 1 \\ L(X) &= \frac{1}{1-X} \end{aligned}$$

## Derivate

$$\begin{aligned} L(X) &= \frac{1}{1-X} \\ L'(X) &= \frac{1}{(1-X)^2} \\ L'(X) &= L(X) * L(X) \end{aligned}$$

## Derivate: Alternative syntax

$$\begin{aligned} L &= 1 + X * L \\ \frac{\partial L}{\partial X} &= \frac{\partial}{\partial X} (1 + X * L) = L + X * \frac{\partial L}{\partial X} \\ \frac{\partial L}{\partial X} &= \frac{L}{1-X} = L^2 \end{aligned}$$

Our list zipper exactly! (actually, the context)

```

type ListZ a = (a, CntxL a)
type CntxL a = ([a], [a])
-- or
type ListZipper a = ([a], [a])
-- or
type ListZipper a = ([a], a, [a])
    
```

## Example: Tree Zipper

```
data Tree a = Leaf | Node A (Tree A) (Tree A)
```



## Example: Tree Zipper

```
data Tree a = Leaf | Node A (Tree A) (Tree A)
```

$$T(X) = 1 + X * T^2(X)$$



## Example: Tree Zipper

```
data Tree a = Leaf | Node A (Tree A) (Tree A)
```

$$T(X) = 1 + X * T^2(X)$$

$$T'(X) = T^2(X) + X * 2 * T(X) * T'(X)$$



## Example: Tree Zipper

**data** Tree a = Leaf | Node A (Tree A) (Tree A)

$$T(X) = 1 + X * T^2(X)$$

$$T'(X) = T^2(X) + X * 2 * T(X) * T'(X)$$

$$T'(X) = \frac{T^2(X)}{1 - 2 * X * T(X)}$$



## Example: Tree Zipper

```
data Tree a = Leaf | Node A (Tree A) (Tree A)
```

$$T(X) = 1 + X * T^2(X)$$

$$T'(X) = T^2(X) + X * 2 * T(X) * T'(X)$$

$$T'(X) = \frac{T^2(X)}{1 - 2 * X * T(X)}$$

$$T'(X) = T^2(X) * L(2 * X * T(X))$$

```
type TreeZipper a = (a, TreeContext a)
```

```
type TreeContext a =
```



## Example: Tree Zipper

```
data Tree a = Leaf | Node A (Tree A) (Tree A)
```

$$T(X) = 1 + X * T^2(X)$$

$$T'(X) = T^2(X) + X * 2 * T(X) * T'(X)$$

$$T'(X) = \frac{T^2(X)}{1 - 2 * X * T(X)}$$

$$T'(X) = T^2(X) * L(2 * X * T(X))$$

```
type TreeZipper a = (a, TreeContext a)
```

```
type TreeContext a =
```

```
  (Tree a, -- left subtree of the hole
```

```
   Tree a, -- right subtree of the hole
```

```
  [( -- list of tuples
```

```
    Bool, -- direction we come from: left or right
```

```
    a,    -- value of the parent node
```

```
    Tree a -- another subtree of the parent node
```

```
  ]])
```

```
-- Alternative definition
```

```
type TreeZipper' =
```

```
(
```

```
  Tree a, -- tree in the hole
```

```
  [( -- list of tuples
```

```
    Direction, -- left or right subtree of the parent node
```

```
    a,         -- value in the parent node
```

```
    Tree a     -- another child of the parent node
```

```
  ]])
```





# Outline for section 2

- 1 Zippers
  - Motivation
  - List Zipper
  - How to derive a zipper?
  - Tree Zipper

- 2 “Optics”
  - Lens: Motivation
  - **Control.Lens**
  - More on **Control.Lens**
  - Prism
  - Traversal

# Lens: Motivation

> Consider some basic data type with *getter* and *setter*:

```
data Athlete = Athlete String
```

```
getName :: Athlete -> String
```

```
getName (Athlete name) = name
```

```
setName :: Athlete -> String -> Athlete
```

```
setName (Athlete _) name = Athlete name
```

# Lens: Motivation

> Consider some basic data type with *getter* and *setter*:

```
data Athlete = Athlete String
```

```
getName :: Athlete -> String
```

```
getName (Athlete name) = name
```

```
setName :: Athlete -> String -> Athlete
```

```
setName (Athlete _) name = Athlete name
```

> This works, but it's *tedious*; Let's use *record* syntax instead

```
data Athlete = Athlete { name :: String }
```

```
main :: IO ()
```

```
main = putStrLn nameOfRealAthlete where
```

```
  athleteWithoutName = Athlete ""
```

```
  realAthlete        = athleteWithoutName { name = "Athlete's name" }
```

```
  nameOfRealAthlete = name realAthlete
```

# Lens: Motivation

> Consider some basic data type with *getter* and *setter*:

```
data Athlete = Athlete String
```

```
getName :: Athlete -> String
```

```
getName (Athlete name) = name
```

```
setName :: Athlete -> String -> Athlete
```

```
setName (Athlete _) name = Athlete name
```

> This works, but it's *tedious*; Let's use *record* syntax instead

```
data Athlete = Athlete { name :: String }
```

```
main :: IO ()
```

```
main = putStrLn nameOfRealAthlete where
```

```
  athleteWithoutName = Athlete ""
```

```
  realAthlete        = athleteWithoutName { name = "Athlete's name" }
```

```
  nameOfRealAthlete = name realAthlete
```

> But what happens when we introduce a new data type with the *same* field name?

```
data Athlete = Athlete { name :: String }  
data Club = Club { name :: String }
```

Error: Multiple  
declarations of 'name'

```
data Athlete = Athlete { name :: String }
data Club = Club { name :: String }
```

Error: Multiple  
declarations of 'name'

› Even if we will use different files:

```
-- Athlete.hs
```

```
data Athlete = Athlete { name :: String }
```

```
-- Club.hs
```

```
data Club = Club { name :: String }
```

```
-- Main.hs
```

```
import Athlete
```

```
import Club
```

```
blankAthlete = Athlete { name = "" }
```

```
-- Ambiguous occurrence `name`
```

```
-- It could refer to either `Athlete.name`
```

```
-- or `Club.name`
```

> Ok, let's use aliases

```
-- Main.hs
```

```
module Main where
```

```
import Athlete as A
```

```
import Club     as C
```

```
main :: IO ()
```

```
main = putStrLn $ nameOfRealAthlete ++ ", " ++ nameOfRealClub where
```

```
  athleteWithoutName = Athlete ""
```

```
  realAthlete        = athleteWithoutName { A.name = "A name" }
```

```
  nameOfRealAthlete  = A.name realAthlete
```

```
  clubWithoutName    = Club ""
```

```
  realClub           = clubWithoutName { C.name = "C name" }
```

```
  nameOfRealClub     = C.name realClub
```

> Ok, let's use aliases

```
-- Main.hs
module Main where

import Athlete as A
import Club     as C

main :: IO ()
main = putStrLn $ nameOfRealAthlete ++ ", " ++ nameOfRealClub where
  athleteWithoutName = Athlete ""
  realAthlete        = athleteWithoutName { A.name = "A name" }
  nameOfRealAthlete  = A.name realAthlete
  clubWithoutName    = Club ""
  realClub           = clubWithoutName { C.name = "C name" }
  nameOfRealClub     = C.name realClub
```

> This may work, but ... module number and aliases can grow!



➤ Fine, let's use different names for fields

```
-- Club.hs
module Club where
data Club = Club { clubName :: String }
-- Athlete.hs
module Athlete where
data Athlete = Athlete { athleteName :: String }
-- Main.hs
import Athlete
import Club

main = putStrLn $ nameOfRealAthlete ++ ", " ++ nameOfRealClub where
  athleteWithoutName = Athlete ""
  realAthlete         = athleteWithoutName { athleteName = "A name" }
  nameOfRealAthlete   = athleteName realAthlete
  clubWithoutName     = Club ""
  realClub            = clubWithoutName { clubName = "C name" }
  nameOfRealClub      = clubName realClub
```

➤ Again, works but it is not what we really want

> Let's define a type class instead:

```
class HasName a where
```

```
  getName :: a -> String
```

```
  setName :: String -> a -> a
```

```
instance HasName Athlete where
```

```
  getName athlete = athleteName athlete
```

```
  setName newName athlete = athlete { athleteName = newName }
```

```
instance HasName Club where
```

```
  getName club = clubName club
```

```
  setName newName club = club { clubName = newName }
```

```
main = putStrLn $ nameOfRealAthlete ++ ", " ++ nameOfRealClub where
```

```
  athleteWithoutName = Athlete ""
```

```
  realAthlete         = setName "A name" athleteWithoutName
```

```
  nameOfRealAthlete   = getName realAthlete
```

```
  clubWithoutName     = Club ""
```

```
  realClub            = setName "C name" clubWithoutName
```

```
  nameOfRealClub      = getName realClub
```



> Let's get rid of **String**; Maybe someone wants to redefine it

```
{-# LANGUAGE MultiParamTypeClasses, FlexibleInstances #-}
```

```
class HasName a b where  
  getName :: a -> b  
  setName :: b -> a -> a
```

```
instance HasName Athlete Text where
```

```
  getName athlete = athleteName athlete  
  setName newName athlete = athlete { athleteName = newName }
```

```
instance HasName Club String where
```

```
  getName club = clubName club  
  setName newName club = club { clubName = newName }
```

```
main = putStrLn $ unpack nameOfRealAthlete ++ ", " ++ nameOfRealClub where
```

```
  athleteWithoutName = Athlete empty  
  realAthlete         = setName (pack "A name") athleteWithoutName  
  nameOfRealAthlete   = getName realAthlete  
  clubWithoutName     = Club ""  
  realClub            = setName "C name" clubWithoutName  
  nameOfRealClub      = getName realClub
```

> Can we do better? It's functional programming: it should be *brief* and *elegant*

## Finally, Lens

```
{-# LANGUAGE MultiParamTypeClasses, FlexibleInstances #-}

import Data.Text
import Athlete
import Club

data Lens a b = Lens { get :: a -> b
                      , set :: b -> a -> a }

athleteNameLens :: Lens (Athlete a) a
athleteNameLens = Lens { get = \athlete -> athleteName athlete
                       , set = \newName athlete -> athlete { athleteName = newName } }

clubNameLens :: Lens Club String
clubNameLens = Lens { get = \club -> clubName club
                    , set = \newName club -> club { clubName = newName } }

class HasName a b where name :: Lens a b
instance HasName (Athlete a) a where name = athleteNameLens
instance HasName Club String where name = clubNameLens

main = putStrLn $ unpack nameOfRealAthlete ++ ", " ++ nameOfRealClub where
  athleteWithoutName = Athlete empty
  realAthlete        = set name (pack "A name") athleteWithoutName
  nameOfRealAthlete  = get name realAthlete
  clubWithoutName    = Club ""
  realClub           = set name "C name" clubWithoutName
  nameOfRealClub     = get name realClub
```

# Control.Lens

```
{-# LANGUAGE TemplateHaskell, MultiParamTypeClasses, FlexibleInstances #-}

import Control.Lens
import Data.Text

data Athlete a = Athlete { _athleteName :: a }
makeLenses ''Athlete

data Club = Club { _clubName :: String }
makeLenses ''Club

class HasName a b where name :: Lens' a b

instance HasName (Athlete a) a where name = athleteName

instance HasName Club String where name = clubName

main = putStrLn $ unpack nameOfRealAthlete ++ ", " ++ nameOfRealClub where
  athleteWithoutName = Athlete empty
  realAthlete        = set name (pack "A name") athleteWithoutName
  nameOfRealAthlete  = view name realAthlete
  clubWithoutName    = Club ""
  realClub           = set name "C name" clubWithoutName
  nameOfRealClub     = view name realClub
```

## Even More: FunctionalDependencies

```
{-# LANGUAGE TemplateHaskell, MultiParamTypeClasses, FlexibleInstances,  
    FunctionalDependencies #-}
```

```
import Control.Lens  
import Data.Text
```

```
data Athlete a = Athlete { _athleteName :: a }  
makeFields ''Athlete
```

```
data Club = Club { _clubName :: String }  
makeFields ''Club
```

```
main = putStrLn $ unpack nameOfRealAthlete ++ ", " ++ nameOfRealClub where  
    athleteWithoutName = Athlete empty  
    realAthlete         = set name (pack "A name") athleteWithoutName  
    nameOfRealAthlete   = view name realAthlete  
    clubWithoutName     = Club ""  
    realClub            = set name "C name" clubWithoutName  
    nameOfRealClub     = view name realClub
```

`_1`

```
ghci> view _1 (1,2)
1
ghci> view _3 (1,2,3)
3
```

### Composition; infix notation

```
ghci> view (_1 . _2) ((1,2),3)
2
ghci> ((1,2),3) ^. _1
(1,2)
ghci> ((1,2),3) ^. _1 . _2
2
```

### Modification

```
ghci> set _1 3 (1,2)
(3,2)
ghci> set _1 "Hello" (1,2)
("Hello",2)
ghci> over _1 length ("Hello","World")
(5,"World")
```

### Infix notation

```
ghci> _1 .~ "Hello" $ (1,2)
("Hello",2)
ghci> (1,2) & _1 .~ "Hello"
("Hello",2)
ghci> _1 %~ (^2) $ (2,3)
(4,3)
```

`_1`

```
ghci> view _1 (1,2)
1
ghci> view _3 (1,2,3)
3
```

## Composition; infix notation

```
ghci> view (_1 . _2) ((1,2),3)
2
ghci> ((1,2),3) ^. _1
(1,2)
ghci> ((1,2),3) ^. _1 . _2
2
```

## Modification

```
ghci> set _1 3 (1,2)
(3,2)
ghci> set _1 "Hello" (1,2)
("Hello",2)
ghci> over _1 length ("Hello","World")
(5,"World")
```

## Infix notation

```
ghci> _1 .~ "Hello" $ (1,2)
("Hello",2)
ghci> (1,2) & _1 .~ "Hello"
("Hello",2)
ghci> _1 %~ (^2) $ (2,3)
(4,3)
```

## Lens laws

$$\begin{aligned} \text{view } l (\text{set } l \ v \ s) &\equiv v \\ \text{set } l (\text{view } l \ s) \ s &\equiv s \\ \text{set } l \ v' (\text{set } l \ v \ s) &\equiv \text{set } l \ v' \ s \end{aligned}$$



› Prism for sum types is the same as lens for product type

## Examples

```
ghci> preview _Left (Left 1)
Just 1
ghci> preview _Right (Left 1)
Nothing
ghci> review _Left "abc"
Left "abc"
```

➤ Prism for sum types is the same as lens for product type

## Examples

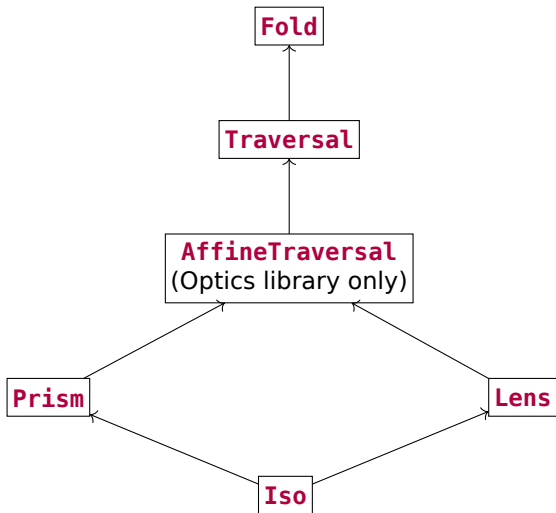
```
ghci> preview _Left (Left 1)
Just 1
ghci> preview _Right (Left 1)
Nothing
ghci> review _Left "abc"
Left "abc"
```

## Composition of Lenses and Prisms

```
ghci> Left (1,2,3) ^? _Left . _2
Just 2
ghci> (Left 1, Left 1, Right "abc")
  ^? _3 . _Right
Just "abc"
ghci> (Left 1, Left 2, Right "abc")
  ^? _3 . _Left
Nothing
```

- Lenses and prisms are closed under composition
- Composition of prisms and lenses is a **Treaversal**
- **Traverse** can have a zero, one or more focuses

# Optics hierarchy



# Traversal

```
data Atom      = Atom      { _element :: String, _point :: Point } deriving (Show)
data Point     = Point     { _x       :: Double, _y       :: Double } deriving (Show)
data Molecule = Molecule { _atoms   :: [Atom]    } deriving (Show)
$(makeLenses ''Atom)
$(makeLenses ''Point)
$(makeLenses ''Molecule)
```

# Traversal

```
data Atom      = Atom      { _element :: String, _point  :: Point } deriving (Show)
data Point     = Point     { _x       :: Double, _y       :: Double } deriving (Show)
data Molecule = Molecule { _atoms   :: [Atom]      } deriving (Show)
$(makeLenses  'Atom)
$(makeLenses  'Point)
$(makeLenses  'Molecule)
```

**Q:** What is a lens?

**A:** a first class getter and setter for a value We could pretend that it is a record with two fields:

```
data Lens a b = Lens
  { view  :: a -> b
  , over :: (b -> b) -> (a -> a)
  }
```

**Q:** What is a traversal?

**A:** first class getter and setter for an arbitrary number of values Think of a traversal as a record with two fields:

```
data Traversal' a b = Traversal'
  { toListOf :: a -> [b]
  , over     :: (b -> b) -> (a -> a)
  }
```

# Traversal

```
data Atom      = Atom      { _element :: String, _point :: Point } deriving (Show)
data Point     = Point     { _x       :: Double, _y       :: Double } deriving (Show)
data Molecule = Molecule { _atoms   :: [Atom]           } deriving (Show)
$(makeLenses ''Atom)
$(makeLenses ''Point)
$(makeLenses ''Molecule)
```

Q: What is a lens?

A: a first class getter and setter for a value We could pretend that it is a record with two fields:

```
data Lens a b = Lens
  { view  :: a -> b
  , over :: (b -> b) -> (a -> a)
  }
```

Q: What is the type of a lens?

```
point :: Lens' Atom Point
x      :: Lens' Point Double
```

Q: What is a traversal?

A: first class getter and setter for an arbitrary number of values Think of a traversal as a record with two fields:

```
data Traversal' a b = Traversal'
  { toListOf :: a -> [b]
  , over     :: (b -> b) -> (a -> a)
  }
```

Q: What is the type of a traversal?

```
atoms :: Traversal' Molecule [Atom]
```

# Traversal

```
data Atom      = Atom      { _element :: String, _point :: Point } deriving (Show)
data Point     = Point     { _x        :: Double, _y         :: Double } deriving (Show)
data Molecule = Molecule { _atoms   :: [Atom]          } deriving (Show)
$(makeLenses ''Atom)
$(makeLenses ''Point)
$(makeLenses ''Molecule)
```

Q: What is a lens?

A: a first class getter and setter for a value We could pretend that it is a record with two fields:

```
data Lens a b = Lens
  { view  :: a -> b
  , over :: (b -> b) -> (a -> a)
  }
```

Q: What is the type of a lens?

```
point :: Lens' Atom Point
x      :: Lens' Point Double
```

The actual definition of `Lens'` is:

```
type Lens' a b =
  forall (f :: * -> *). Functor f =>
    (b -> f b) -> (a -> f a)
  = Lens s s a a
type Lens s t a b =
  forall (f :: * -> *). Functor f =>
    (a -> f b) -> s -> f t
```

Q: What is a traversal?

A: first class getter and setter for an arbitrary number of values Think of a traversal as a record with two fields:

```
data Traversal' a b = Traversal'
  { toListOf :: a -> [b]
  , over     :: (b -> b) -> (a -> a)
  }
```

Q: What is the type of a traversal?

```
atoms :: Traversal' Molecule [Atom]
```

The actual definition of `Traversal'` is:

```
type Traversal' a b =
  forall (f :: * -> *). Applicative f =>
    (b -> f b) -> (a -> f a)
  = Traversal s s a a
type Traversal s t a b =
  forall (f :: * -> *). Applicative f =>
    (a -> f b) -> s -> f t
```

## Traversal: Example

```
data Atom      = Atom      { _element :: String, _point :: Point } deriving (Show)
data Point    = Point     { _x        :: Double, _y        :: Double } deriving (Show)
data Molecule = Molecule { _atoms   :: [Atom]       } deriving (Show)
$(makeLenses ''Atom)
$(makeLenses ''Point)
$(makeLenses ''Molecule)

shiftAtomX :: Atom -> Atom
shiftAtomX = over (point . x) (+ 1)

shiftMoleculeX :: Molecule -> Molecule
shiftMoleculeX = over (atoms . traverse . point . x) (+ 1)

main =
  let atom1 = Atom { _element = "C", _point = Point { _x = 1.0, _y = 2.0 } }
      atom2 = Atom { _element = "O", _point = Point { _x = 3.0, _y = 4.0 } }
      molecule = Molecule { _atoms = [atom1, atom2] }
  in do
    print $ shiftAtomX atom1
    print $ shiftMoleculeX molecule

-- Atom { _element = "C", _point = Point { _x = 2.0, _y = 2.0 } }
-- Molecule { _atoms = [Atom { _element = "C", _point = Point { _x = 2.0, _y = 2.0 } },
--                        Atom { _element = "O", _point = Point { _x = 4.0, _y = 4.0 } } ] }
```



# Consuming Lenses and Traversals

```
view :: Lens' a b -> a -> b
over :: Lens' a b -> (b -> b) -> a -> a

set  :: Lens' a b ->      b -> a -> a
set lens b = over lens (\_ -> b)
---
over :: Traversal' a b -> (b -> b) -> a -> a

set  :: Traversal' a b ->      b -> a -> a
set traversal b = over traversal (\_ -> b)

toListOf :: Traversal' a b -> a -> [b]
```

## Operators

prefix	infix
<code>view</code> <code>_1</code> (1,2)	(1,2) <code>^.</code> <code>_1</code>
<code>set</code> <code>_1</code> 7 (1,2)	( <code>_1</code> <code>..</code> 7) <code>^.</code> (1,2)
<code>over</code> <code>_1</code> (2 *) (1,2)	( <code>_1</code> <code>\%~</code> (2 *)) <code>^.</code> (1,2)
<code>toListOf</code> traverse [1..4]	[1..4] <code>^..</code> traverse
<code>preview</code> traverse []	[] <code>^?</code> traverse

The End